I

# **Georock 3D**

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# 1 Georock 3D

**GeoRock 3D** is a program for three dimensional analysis of rock fall and for the design and optimisation of protective works utilising a sophisticated algorithm for spatial analysis.

### Slope modelling

The slope in question is generated from an xyz mesh that may be imported from formats or suitable software. The triangulation technique is extremely important and it is recommended that Incremental Delaunay or anysotropic triangulation are used.

In this respect the Geostru software "Trispace" is eminently suitable.

A tridimensional model can be imported directly from chart work using Geostru Autocad Application (G.A.A.)

## Terrain properties

The slope is discretized in homogeneous zones having similar mechanic characteristics, namely normal and tangential restitution coefficients.

#### Block characteristics

Geometric characteristics are specified for a spherical block rotating around its center of gravity, and mechanicaly its hardness and mass.

#### Launch sites

Launch sites can be indicated directly on the 3D model specifying, for each launch, the launch velocity.

#### Calculation

This provides three-dimensional trajectories, speed and energy.

#### Protective works

Protective works can be inserted directly on the 3D model and their position optimized in relation to the displayed trajectories.

## Barrier archive

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Facilities are provided for the archive of standard barriers that may be reutilised directly on the 3D model.

# Protective works sizing

Geostru Software provides a tool, BARRIER DESIGN (B.D.) for the design a verification of rigid or elastic barriers.

## Interfaces

This software interfaces with Geostru softwares Trispace and Georock 2D.



# 1.1 Home

FILE	HOME	VIEW	TOOLS	CALCULATION	I OUTPL	IT ?									
General	Personal data	GEO Geo Drop Box	Import elevation attributed plane	Import 3D model	Material management	Apply material to individual element	Apply material to a region	Generate launch area	Launch area properties	Add single block	Delete blocks	Insert barrier	Edit barrier	Insert Edit marker markers	
						n	ata								

# 1.1.1 GeoDropBox



User can connect their GeoDropBox account to upload and save files directly from the software

Geo	x DropBox
Email	
Password	$\bigcirc$
Ricordami	Password dimenticata?
Logi	n
Non hai ancora un account?	Registrati

### 1.1.2 General and Personal Data



In General Data and Personal Data it is possible to enter the project data and the professional's Master Data. This data will then be printed in the output report.

#### 1.1.3 Elevation attributed plane

**Georock 3D** is based on the three-dimensional definition of the surface through a dimensioned plane.

Defining a correctly quoted plan is indispensable as it is the predominant data of the analysis.

Therefore, it is advisable to use software for the generation of the plan quoted with triangulation techniques of the incremental **Delaunay** type or with **anisotropic triangulation**.

The **Trispace** software produced by Geostru software is a tool specially created for 3D analysis of the territory, which allows you to generate

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sophisticated meshes with many analysis options, including incremental and anisotropic triangulation.

It is also possible to extract three-dimensional points from 3D cartographies through **G.A.A (Geostru Autocad Applications)** which can be triangulated with dedicated software.

Also, the reading format of the quoted plane is text, each line contains the coordinates of the triangle in the format **[x,y,z] [x,y,z] [x,y,z]**.

### 1.1.3.1 Import of an elevation attributed plane



Before developing a project the import of information related to the triangulation of the work area must be done. GeoRock 3D is perfectly compatible with the triangulations generated using TriSpace, and these triangulations can be imported using the proper command in Data menu.

## 1.1.4 Materials assignment



The software offers the facility of a personalized database which gathers a wide range of materials that can be used for different regions of the triangulation.

For every material is necessary to define the Rn and Rt restitution parameters , the description and a color that allows the identification of the area.

A material can be assigned to a single element or to an entire region.

# Single element



To assign a material to a single element choose the appropriate tool, click with the mouse on the element to alter and select a material from the pop-up window.

## N.B.

To assign the same material to other single elements press the "Ctrl" key while selecting the elements.

Description	Normal restitution coefficient	Tangential restitution coefficient	Color
Mat 1	0.80	0.80	
Mat 2	0.80	0.80	
Mat 3	0.80	0.80	10
Mat 4	0.80	0.80	
Mat 5	0.80	0.80	
Mat 6	0.80	0.80	
Mat 7	0.80	0.80	
Mat 8	0.80	0.80	
Mat 9	0.80	0.80	
Mat 10	0.80	0.80	
Mat 11	0.80	0.80	
Mat 12	0.80	0.80	
Mat 13	0.80	0.80	
Mat 14	0.80	0.80	
Mat 15	1.00	1.00	
Mat 16	0.30	0.30	
Prova	1.00	1.00	

Region

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To assign a material to a group of elements choose the appropriate tool, select an area from the graphic, click with the mouse and choose a material from the pop-up window.

## Valori dei coefficienti di restituzione in letteratura

TEDICTION

Valori ricavati da prove in sito da Pfeiffer e Bowen (1990)

E DEL PENDIO	Kn	Kt
SMOOTH SURFACES OF RESISTANT MATERIAL SUCH AS ROAD OR BOCKY SUBFACE	0.37 – 0.42	0.87 – 0.92
PREVALENCE OF INTACT BEDROCK WITH ISOLATED BLOCKS DETRIT	0.33 – 0.37	0.83 – 0.87
CONSISTING OF LARGE BLOCKS WITH LITTLE AFFIORATING VEGETATION	0.30 – 0.33	0.82 – 0.85
DETRIT COVERED BY ABUNDANT VEGETATION AND SPARSELY VEGETATED SOIL	0.28 - 0.30	0.80 - 0.83
SLOPE IN THE EARTH.		0.78 - 0.82
	Richards, 1988	
CHARACTER ISTICS OF	Kn Kt	BIBLIOGRA PHIC

THE SLOPE			REFERENCE
AFFLORATIN G ROCK	0.90 - 0.80	0.75 – 0.65	
DETRIT AND	0.80 0.50	0.65 0.45	
G ROCK SOLID	0.80 - 0.50	0.65 - 0.45	PITEAU & CLAYTON
DETRIT AND ROCK	0.50 - 0.40	0.45 - 0.35	
VEGETATION ROCKY	0.40 - 0.20	0.30 - 0.20	
SUBSTRATE (BEDROCK)	0.53	0.99	HOEK (RAPPORTO
ROAD SURFACE	0.40	0.90	NON PUBBLICATO
AFFLORATIN G ROCK	0.35	0.85	PER I "DEPARTMEN
DETRIT	0.32	0.82	TS OF
DETRIT AND VEGETATION	0.32	0.80	TRANSPORTA TION
LITTLE	0.30	0.80	1987)

## 1.1.5 Points file import (x, y, z)

The geometry of the slope can be imported from text files with specified coordinates x,y,z of each point organized in 3 fields (columns): each record (row) indicates a point.

The coordinates to be imported must be flat, that is, expressed in m or other flat size and not in latitude and longitude or angles.

The import command can be reached from "Home">"Import 3D model (point files (x,y,z))":

📓 Geor	ock 3D														
🗋 📁		) 🥖 🛍	min ∓												
FILE	HOME	VIEW	TOOLS	CALCULATIO	N OUTPU	IT ?									
	:	GDB	30	×71 + + •		0	C	3		69	P		圖		
General data	Personal data	Geo Drop Box	Import elevation attributed plane	Import 3D model	Material management	Apply material to individual element	Apply material to a region	Generate launch area	Launch area properties	Add single block	Delete blocks	Insert barrier	Edit barrier	Insert Edit marker markers	
						D	ata								
Trajecto	ies Heig	hts Energy													

The reading window of the text file containing the coordinates of the points will be opened:

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Import points			
📄 New 🧀 Open 🛛 📀 Assign Geo	rock 3D		
	File Extraction         Select           Separator         :	 	
	2 4 V Extract [X] [Y] [Z]		
	Z Mesh		

Where in the top bar we can find:

- New : allows re-initialization of the import by deleting the loaded data;
- **Open** *—* : Opens the opening file dialog;
- **Assign Georock3D**: after triangulation, if the result is the desired one, the geometry is assigned to the software;
- The import window can be divided into three areas:



- 1. File Preview: this area shows a preview of the text file to be opened;
- 2. **File extraction**: here you can set import parameters such as the field separator (columns), the number of the row from which to start the

extraction, and the numer of the field (column) of the X, Y, Z coordinates. Pressing Extract will bring up another dialog box to filter the data:

Filter	×	
Escludere tutte le X con valore pari a Escludere tutte le Y con valore pari a Campo di validità delle quote min, max	■ □ □ □ ○ K .::	Exclude all X with a value equal to Exclude all Y with a value equal to Validity range of quotas min.max

The Mesh button starts the triangulation of the points and Stop stops it; 3. **3D preview**: The 3D preview of the triangulation is displayed.

Here is a short tutorial on how to import geometry from a text file:



punti di un versante.

1.1.6 Selecing a launch area



The launch site of the block is defined by selecting, with the mouse, an area from the graphic.

For this area the number of blocks to analyse, along x and y directions, in a positive direction, must be defined.

The software will provide the automatic placing of the blocks in the selected area, considering as elevation the plane elevation in the application point.



## 1.1.7 Initial conditions

The program allows the personalization of the launch position, the definition of the initial velocity components along the coordinate axis, assigning the density and the diameter for every block subjected to trajectory computation.

	Block deta	ils								
e these commands to modify the ues for all blocks	N°	<mark>X [m]</mark>	Y [m]	Z [m]	Vx [m/s]	Vy [m/s]	Vz [m/s]	Diameter [m]	Density [kg/m3]	Color
	1	8,643.32	19,823.01	32.68	0.00	0.00	0.00	1.00	2,000.00	
	2	8,636.53	19,823.92	33. <mark>4</mark> 1	0.00	0.00	0.00	1.00	2,000.00	
	3	8,630.26	19,826.27	33.68	0.00	0.00	0.00	1.00	2,000.00	
ssegna velocità	4	8,623.65	19,829.39	34.03	0.00	0.00	0.00	1.00	2,000.00	
x [m/s]	5	8,616.28	19,833.28	34.15	0.00	0.00	0.00	1.00	2,000.00	
/ [m/s]	6	8,605.28	19,841.51	33.88	0.00	0.00	0.00	1.00	2,000.00	
7 [m/e]	7	8,598.78	19,848.88	34.32	0.00	0.00	0.00	1.00	2,000.00	
r fur al	8	8,598.60	19,852.47	33.43	0.00	0.00	0.00	1.00	2,000.00	
Z	9	8,603.55	19,852.29	30.67	0.00	0.00	0.00	1.00	2,000.00	
	10	8,613.73	19,845.97	29.09	0.00	0.00	0.00	1.00	2,000.00	
×	11	8,616.68	19,842.36	30.19	0.00	0.00	0.00	1.00	2,000.00	
Ok	12	8,621.59	19,835.09	31.08	0.00	0.00	0.00	1.00	2,000.00	
OK	13	8,630.13	19,829.88	31.57	0.00	0.00	0.00	1.00	2,000.00	
segna diametro	14	8,642.78	19,828.47	30.00	0.00	0.00	0.00	1.00	2,000.00	
[m]	15	8,639.10	19,832.99	28.40	0.00	0.00	0.00	1.00	2,000.00	
	16	8,629.72	19,838.01	28.25	0.00	0.00	0.00	1.00	2,000.00	
Ok	17	8,625.14	19,840.01	28.57	0.00	0.00	0.00	1.00	2,000.00	
ssegna densità										
ensità [kg/m3]										

#### 1.1.7.1 Calculation of Initial Velocity using the Impulse Theorem

The impulse theorem states that the impulse exerted on an object is equal to the change in the object's momentum. In formula, we can write it as:

$$I = \Delta p$$

Where:

- I is the impulse.
- $\Delta p$  is the change in momentum.

Momentum p is defined as the product of mass m and velocity v:

If we know the force Fapplied to the mass and the time t during which this force is applied, the impulse I can be calculated as:

$$I = F * t$$

To find the initial velocity  $v_0$ , we can express the change in momentum as:

$$\Delta p = m * v - m * v_0$$

Where:

- v is the final velocity.
- $v_0$  is the initial velocity.

Using the impulse theorem:

 $F * t = m * v - m * v_0$ 

From which we can isolate the initial velocity  $v_0$ :

$$v_0 = v - (F * t) / m$$

# **Example Calculation:**

Suppose we have a mass with the following characteristics:

- Mass m = 10 kg
- A constant force F = 50 N is applied for t = 4 s
- Final velocity v = 30 m/s

Let's calculate the initial velocity  $v_0$ :

1. Calculate the impulse:

$$I = F * t = 50 N * 4 s = 200 Ns$$

2. Using the formula to find  $v_0$ :

$$v_0 = v - (I / m)$$
  
 $v_0 = 30 m/s - (200 Ns / 10 kg)$   
 $v_0 = 30 m/s - 20 m/s$   
 $v_0 = 10 m/s$ 

Therefore, the initial velocity of the mass was 10 m/s.

## 1.1.8 Barriers assignment



Using the mouse, the barriers can be inserted in a spatial arrangement. Every barrier is formed by one ore more segments, each one having its own color, description, height, inclination and a maximum absorption energy.

The barrier position can be modified with the mouse directly on the graphic.



After defining the barrier, the last segment must be confirmed with a click on the right mouse button.

Barrier properties		×
Barriers list	Barrier properties	
Barrier n° 1 Barrier n° 1 Segment no. 1 Barrier n° 2 Barrier n° 2 Barrier n° 2 Segment no. 1 Segment no. 2 Segment no. 2 Segment no. 3 Segment no. 3 Segment no. 4	✓         Geometry Angle [*]         90           Barrier segment height [m]         3           >         Point P1         8649.02, 19834.93, 2           >         Point P2         8642.04, 19838.58, 2           ✓         Property         Color          Color         139, 0, 0           Description         Descrizione           Maximum energy [KJ]         10000	4.69 4.51
	Angle [°] Barrier angle [°]. (by default the barrier angle is assigned equiper 90°)	ual to

## 1.1.9 Markers

Markers



You can insert cylindrical elements as references to the area. Once inserted through the tool "Edit markers" you can change the coordinates, radius, height and the color.

([m]	Y [m]	Z [m]	Radius [m]	Height [m]	Color
8,650.37	19,835.78	24.62	0.50	1.00	
8,633.80	19,843.39	24.55	0.50	1.00	
8,647.57	19,858.23	14.24	0.50	1.00	
8,627.94	19,867.93	14.33	0.50	1.00	

## 1.2 View

Thickness lines trajectories 2	Fit Zoom window	Pan	Gradient	<b>₽</b> Rotate	Тор	Bottom	Left	Front	Back	Wire	Render	Shade	Shade On
Graphics	View			3D View									

From the View menu you can use all those tools for graphical visualization of the 3D model.

## 1.2.1 Raster image

A raster image can be inserted in the project only after importing the elevation attributed plan.

The import procedure starts with the choice of the jpg image file that contains the representation to be included in the project. The calibration of the image to use starts through a guided procedure when the "**Start Calibration**" button is pressed. The user is asked to select with the mouse two points on the map with known coordinates, expressed in same reference system used for the elevation attributed plan.

#### ATTENTION:

The coordinates must be expressed in the same reference system used for the elevation attributed plan.

To obtain a correct calibration of the map is necessary to select two points not close to each other and arranged diagonally.

Through the "Tools" menu of the main window the user is given the option to delete the imported image, to re-import or view the properties.

We included an import example in the **Raster Import**. The digital model has been realized through GeoStru Maps.

## 1.2.2 Gradient



The Gradient option allows you to view the mesh based on 4 different options:

- Color according to the material (user-defined lithology);
- Color according to altitude;
- Transparent elements (transparent mesh);
- Raster surface (if a raster image is loaded).

# 1.3 Instruments



1.3.1 Crop digital plan quoted



With this option, selecting an area in the mesh will delete all items outside it:





### 1.3.2 Section



To generate a section of the work area select the proper tool and define with the mouse a section line.

At the ending of the selection, the section will be displayed in a separate window and it can be exported in edp format.

The sections can be saved in edp format.

The edp format is an export format for the GeoStru software files. This type of files can be read by GoeRock 2D, Slope, etc.



# 1.4 Computation, Graphs

Once the input phase is done, the software is able to determine the blocks motion as a function of the impacts on the soil and on eventual barriers previously defined. The trajectories are displayed on the graphic as they are calculated by the program. For every trajectory information as block initial position, minimum and maximum velocity values and absolute position are available. The user can choose to view only the trajectories he is interested in. By scrolling with the mouse on a trajectory, the block's elevation, the velocity (as a module) and the energy in that point are displayed.





# 1.4.1 Animation

Slow	*	Animation
	Animation	

Besides the tridimensional static view of the trajectory, calculated for every block, an animated view can be displayed.

The animation can be interrupted, speed up or slow down.



# 2 Geoapp

# Geoapp: the largest web suite for online calculations

The applications present in <u>Geostru Geoapp</u> were created to support the worker for the solution of multiple professional cases.

Geoapp includes over 40 <u>applications</u> for: Engineering, Geology, Geophysics, Hydrology and Hydraulics.

Most of the applications are **free**, others require a monthly or annual **subscription**.

Having a subscription means:

- access to the apps from everywhere and every device;
- saving files in cloud and locally;
- reopening files for further elaborations;
- generating prints and graphics;

- notifications about new apps and their inclusion in your subscription;
- access to the newest versions and features;
- support service throught Tickets.

# 2.1 Geoapp Section

# General and Engineering, Geotechnics and Geology

Among the applications present, a wide range can be used for GeoRock. For this purpose, the following applications are recommended:

- Anchored mesh systems
- Nailing passive bars
- Rigid and elastic rockfall barriers
- ≻ <u>Tiranti</u>
- Analysis stability of flat surfaces
- Slips along a plane
- ➢ 3D Wedge

# **3** Software Geomechanical rocks

**GEOSTRU software** – among the most successful software worldwide – highly advanced software products and services in the following fields: civil engineering, geotechnical engineering and geology, **geomechanics software**, geophysics, in situ soil testing, hydrology and hydraulics, topography.

The software is characterized by an intuitive interface and face every single geological problem in 2D-3D environment.

The main suite includes a number of geology programs, fully integrated with each other, characterized by a very competitive cost and a quality guaranteed by over 30,000 installations. The software is widespread in more than 135 countries around the world.

The software GEOSTRU is very easy to use, each software solves a specific problem, reducing the input data to the strictly necessary. Furthermore, being integrated with each other, you have the possibility to export data from/and for each software in the suite.

All products are fully compatible with the Eurocodes, in particular EC7 for geotechnical design, EC8 for the seismic design, EC2/3 for structural design and checks and also allow user custom parameters.

The complete suite for Geomechanics of rocks can be viewed here: Geostru geomechanics software - Software for rocks mechanics

- Geomechanical survey;
- Rock classification;
- Stability analysis;
- 2D and 3D rockfall analysis;
- Rock consolidation

# 4 Rock falls 2D – GeoRock 2D

![](_page_23_Figure_8.jpeg)

GeoRock 2D is a software made by GeoStru for the 2D simulation of rockfalls using **Lumped Mass and C.R.S.P. models**.

The computation assumptions for the Lumped Mass model are plan outline, the slope profile similar to a broken line consisting of straight line segments, point boulder and neglectable air resistance.

**CRSP** (Colorado Rockfall Simulation Program) model has been developed by Pfeiffer and Bowen (1989) to model the falling motion of boulders having the shape of spheres, cylinders or discs, with a circular crosssection in the vertical plane of the movement. The model's reliability was verified by comparisons between numerical results and the results obtained from in situ tests.

### Rock falls 2D.

GeoRock 2D is a software for the rock falls 2D simulation using Lumped Mass and C.R.S.P. models.

The software is interfaced with other GeoStru programs: TriSpace for automatically generating sections from an XYZ plane or raster images and GeoStru Maps, the new online App from GeoStru, that allows you to create topographic sections in a very simple way.

The fall trajectories can be divided into the following areas:

transit area and stop area of 70% of the blocks;

- stop area of 95% of the blocks;
- stop area of 100% of the blocks.

Frequency not exceeding: 5, 50, 89, 95, 100%.

These areas are then assigned the relative hazard classes.

## DATA INPUT

- Graphic input with the mouse (or another pointing device);
- Import from Excel;
- Import of DXF files;
- Import of topographic profile generated by TriSpace;
- Import of raster images;
- Automatic generation of topographic sections to be analyzed using GeoStru Maps.

### **CALCULATION METHODS**

- CRSP method (Colorado Rockfall Simulation Program;
- LUMPED MASS method.

#### WORKS OF INTERVENTION

• Barriers.

#### **GRAPHIC AND COMPUTATION OPTIONS**

- Calculation type: statistical, deterministic;
- Boulder form: Spherical, Cylindrical, Disc;
- Function boulder attached to the slope;
- Function barrier attached to the slope;
- Point launch or definition of the launch area;
- Automatic identification of launch points;
- Animated display of bounce and impact;
- Display of all calculated trajectories;
- Selection of generic trajectory;
- Energy, Velocity, Flight time, and Parabola height information at any point of the trajectory;
- Display of kinematic parameters relative to the barriers;
- Display of percentage of boulders passing defined points on fall path;
- Georock 2D interfaces with Georock 3D for the analysis of threedimensional rock fall.

#### **STANDARD**

UNI 11211/2012 (NTC 2008) and UNI 11211/2018 (NTC 2018) standards with the possibility of defining the various partial safety coefficients for both the Maximum Energy Level (MEL) and the Service Energy Level (SEL); The resistance of the rockfall barriers (kinetic energy) that can be defined for the two energy levels MEL and SEL;

# More information: Rock falls 2D - Georock 2D

# 5 Theoretical notes

The model used for the computation of the trajectories of rock falls considers the blocks like point block that have an impact on an elevation attributed plane. This plane is formed by a grid of tridimensional nodes that form a triangular mesh which represents the entire area between the launch and the stopping of the blocks.

The elements of the grid must be defined so that, in the inside of the perimeter, the inclination and the direction of the slope together with the physical parameters of the model ( the normal and tangential restitution parameters), defined as ratio of the post and pre - impact energy, can be considered constants.

The triangles must be dimensioned right because they can be considered "big" regarding the volume of the blocks and "little" regarding the area involved.

The definition of the launch area assumes a previously analysis of the launch niche so that for every block it can be defined a launch velocity as a function of the initial route along the wall.

This model also carries out the analysis for the positioning and sizing of protective works. The initial computation verifies that the block knocks against the barrier and does not climb over it, so it verifies that the impact kinetic energy of the block can be completely absorbed by the protective work.

This model for the computation of trajectories of rock falls is mostly used for the design of embankments as protective works because, thanks to their high expansion, the embankments can be able to intercept a great number of blocks before requiring maintenance interventions or reconstruction, and assure a remarkable absorption energy, typically generated by long fall routes.

For the design of more precise works, like rock fall nets, the studies on the fall route of blocks are more accurate in the two-

dimensional models that are more sophisticated in the description of the physical phenomenon.

The trajectory of the block can be determined by using the motion equations of a block. Referred to a cartesian orthogonal axis system this equations are:

 $s = v \times t + s_0$  $z = -1/2 \times g \times t^2 + v \times t + z_0$ 

where:

v = block's velocity; t = time; g = gravitational constant; s = crossed space;

In this way the trajectory of the motion results to be formed by a series of parabolas drawn from the launch point to the point where the block knocks first the slope, in the initial phase of the motion, and between two consecutive impact points on the slope, or at the foot, until the stopping point.

By indicating with  $v_n$  and  $v_t$  the components (normal and tangential) of the velocity before the impact,  $v'_n$  and  $v'_t$  after the impact can be calculated using the following relations:

$$v'_n = v_n \times \lambda_n$$
  
 $v'_t = v_t \times \lambda_t$ 

where:

 $\lambda_n$  and  $\lambda_t$  are the restitution coefficients, varying in the interval 0-1

# 5.1 Lumped Mass Method

The assumptions of the Lumped Mass model are:

- a plane outline, a slope profile similar to a broken line consisting of straight segments;
- 2. point block and negligible air resistance.

In this case the trajectory of the block can be determined using the equations of the motion of a body

![](_page_27_Figure_1.jpeg)

The motion is characterized by a constant acceleration a=g=-guy and the initial conditions are v=v0 at time t=0, the moment of launch. From the definition of acceleration in plane motion we obtain the following relationship:

$$\vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}(t) \cdot dt = \vec{v}_0 - gt \cdot \vec{u}_y$$

like

$$\vec{v}(t) = \vec{v}_0 \cos\theta \cdot \vec{u}_x + \vec{v}_0 \sin\theta \cdot \vec{u}_y - gt \cdot \vec{u}_y$$

the speeds of the motions projected on the axes are:

Referring to the same system of orthogonal Cartesian axes the hourly laws of the projected motions are:

$$x = v_x \cdot t + x_0$$
  

$$y = -\frac{1}{2} \cdot g \cdot t^2 + v_y \cdot t + y_0$$
(1)

where:

**vx** horizontal component of the block speed;

vy vertical component of the block speed;

t time;

**g** acceleration of gravity;

 $\boldsymbol{x0}$  abscissa of the point where block detaches from the slope or bumps in the fall motion;

**y0** ordinate of the point where the block is detached from the slope or bumps in the fall motion.

Along the x-axis the motion is uniform, along the y-axis uniformly accelerated.

![](_page_29_Figure_1.jpeg)

In this way the trajectory of the motion of the block is composed of a series of parabolas drawn between the point where the detachment occurs and the point where the block hits the slope for the first time, in the initial phase of the motion, and between two subsequent points of impact on the slope, or at the foot, afterwards, until the end point of arrest.

The coordinates of the points of impact and the components of the velocity are determined by solving the system between equation (1) and the equation of the straight lines which representing the slope profile.

In practice, one proceeds from the point where the block is detached and solve this system of equations by considering from time to time the different equations of the straight lines that contain the successive segments of the broken line up until find the coordinates of a point, impact point, which belongs to the parabola representing the trajectory and falls within one of the segments of the broken line and is therefore also a point of the slope.

This point represents the first point of impact of the block on the slope. The procedure is repeated starting from this point to determine the next arc of the trajectory and a new point of impact.

The loss of kinetic energy due to friction and impact can be modeled by reducing the speed of the falling block each time it hits the slope.

In particular, indicating with vn and vt the components (normal and tangential) of the velocity before the impact, after the impact v'n, v't can be calculated by means of the relations:

$$v'_n = v_n \cdot R_n$$
  
 $v'_t = v_t \cdot R_t$ 

Rn and Rt are called variable coefficients of restitution in the interval 0-1

# 6 Contact

GeoStru Software

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