RockLab

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1.1 Hoek-Brown failure criterion

Hoek-Brown failure criterion

Hoek and Brown have introduced their failure criterion in an attempt to provide analysis data for the design of underground excavations in hard rock. The criterion was derived from the results of Hoek's research (1968) on the brittle fracture of intact rock and Brown's studies (1970) on the model of the jointed rock mass behavior.
The criterion started from the properties of the intact rock and introduced factors to reduce these properties based on the characteristics of joints in a rock mass. The authors have tried to link the empirical criterion to geological observations by means of one of the rock mass classification schemes available and, to this end, they chose the classification proposed by Bieniawski (1976).
Due to the lack of suitable alternatives, the criterion was soon adopted by the community of rock mechanics and its use quickly spread beyond the original limits used in deriving the relations for strength reduction.
As a result, it has become necessary to review these reports and from time to time introduce new elements because of the wide range of practical problems in which the criterion has been applied. Typical of these improvements have been the introduction of the concept of Hoek and Brown (1997) "undisturbed" and "disturbed" rock masses, and the introduction of a modified criterion to impose to zero the tensile strength of the rock mass for the masses of very poor quality (Hoek, Wood and Shah, 1992).

One of the first difficulties arose from the fact that many geotechnical problems, in particular problems of slope stability are more conveniently discussed in terms of shear and normal stresses rather than with the relationships of the mains stress of the original Hoek-Brown criterion, defined by the equation:

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m\frac{\sigma_3'}{\sigma_{ci}} + s\right)^{0.5} \qquad [1]$$

where σ_1' e σ_3' are respectively the major and minor effective stress at main failure σ_{ci} is the uniaxial compressive strength of the intact rock material and m and s are material constants, where s = 1 for intact rock.

An exact relationship between equation 1 and the normal and tangential failure stresses was obtained Hoek and Bray (1981) and later by Ucar (1986) and Londe (1988).

Hoek has discussed the derivation of equivalent friction angles and cohesive forces for a variety of practical situations. These derivations were based on the tangent to the Mohr envelope obtained by Bray (1981). Hoek suggested that the cohesive strength determined by inserting a tangent to Mohr's curvilinear envelope is an upper limit value and can give optimistic results in the stability calculations. Consequently, an average value, determined by the insertion of a linear relationship Mohr-Coulomb with the method of least squares, may be more appropriate. In this work Hoek has also introduced the concept of Hoek-Brown Generalized Criterion in which the shape of the plane of the main stress or Mohr's envelope could be changed by means of a variable coefficient a instead of the term of the square root in equation 1.

Hoek and Brown have tried to consolidate all previous improvements in a full presentation of the failure criterion and have made a number of concrete examples to illustrate its practical application.

In addition to changes in the equations, it was also recognized that the classification of the rock mass of Bieniawski was no longer adequate as a vehicle for the relationship between the failure criterion and the geological observations in the field, particularly for very weak rock masses. This led to the introduction of the GSI index fo Hoek, Wood and Shah (1992), Hoek (1994) and Hoek, Kaiser and Bawden (1995). This index was subsequently extended for weak rock masses in a series of articles by Hoek, Marinos and Benissi (1998) and Marinos and Marinos and Hoek.

The Generalized Criterion of Hoek-Brown

Is expressed as

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m_b\frac{\sigma_3'}{\sigma_{ci}} + s\right)^a \qquad [2]$$

where *mb* is a reduced value of the material constant *mi* and is given by

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right) \quad [3]$$

s and *a* are constants for the rock mass:

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right) \quad [4]$$

$$a = \frac{1}{2} + \frac{1}{6}\left(e^{-GSI/15} - e^{-20/3}\right) \quad [5]$$

D is a factor that depends on the disturbance degree to which the rock mass is subjected by blast damage and stress relaxation. It varies from 0 for undisturbed in situ rock masses to 1 for very disturbed rock masses (see Guidelines for the estimation of the disturbance D).

The uniaxial compressive strength is obtained by setting in equation [2]

$$\sigma_3' = 0$$

providing

$$\sigma_c = \sigma_{ci} \cdot s^a \quad [6]$$

and the tensile strength is given by:

$$\sigma_t = -\frac{s\sigma_{ci}}{m_b} \quad [7]$$

The equation [7] was obtained imposing

$$\sigma_1' = \sigma_3' = \sigma_t$$

in the equation [2]. This represents a condition of the biaxial stress. Hoek has shown that, for brittle materials, the uniaxial tensile strength is equal to the biaxial tensile strength.

Note that the '*transition*' to GSI = 25 for the coefficients *s* and *a* was eliminated in equations 4 and 5 which give continuous uniform transitions throughout the entire range of GSI values. The numerical values of *a* and *s*, given by these equations, are very similar to those given by the previous equations.

The normal and shear stresses are related to the principal stresses through the equations published by Balmer.

$$\sigma_n' = \frac{\sigma_1' + \sigma_3'}{2} - \frac{\sigma_1' - \sigma_3'}{2} \cdot \frac{d\sigma_1'/d\sigma_3' - 1}{d\sigma_1'/d\sigma_3' + 1}$$

$$\tau = (\sigma_1' - \sigma_3') \cdot \frac{\sqrt{d\sigma_1'/d\sigma_3'}}{d\sigma_1'/d\sigma_3' + 1} \quad [9]$$

$$d\sigma_1'/d\sigma_3' = 1 + am_b (m_b \sigma_3'/\sigma_{ci} + s)^{a-1} \quad [10]$$

Deformation modulus

The deformation modulus of the rock mass is given by:

$$E_m(GPa) = \left(1 - \frac{D}{2}\right) \sqrt{\frac{\sigma_{ci}}{100}} \cdot 10^{((GSI-10)/40)} \quad [11a]$$

The equation 11a is valid for

$$\sigma_{ci} \leq 100MPa$$

For

$$\sigma_{ci} > 100MPa$$

is used the equation 11b.

$$E_m(GPa) = \left(1 - \frac{D}{2}\right) \cdot 10^{((GSI-10)/40)} \quad [11b]$$

Note that the original equation proposed by Hoek and Brown has been changed, with the inclusion of the factor D, to allow the effects of the explosion damage and the stress relaxation.

1.2 Mohr-Coulomb criterion

Since most of the geotechnical software is still written in terms of Mohr-Coulomb failure criterion, it is necessary to determine equivalent friction angles and cohesive forces for each rock mass and stress range. This is done by inserting a linear average relationship to the generated curve, solving equation [2] for a range of values of the minor principal stress defined by

$$\sigma_t < \sigma_3 < \sigma'_{3\max}$$

as illustrated in figure 1.

The adaptation process involves balancing the areas above and below the plane of Mohr-Coulomb. This implies the following equations for the angle of friction ϕ' and the cohesive force c':

$$\phi' = \sin^{-1}\left[\frac{6am_b\left(s + m_b\sigma'_{3n}\right)^{a-1}}{2(1+a)(2+a) + 6am_b\left(s + m_b\sigma'_{3n}\right)^{a-1}}\right] \qquad [12]$$

$$c' = \frac{\sigma_{ci}\left[(1+2a)s + (1-a)m_b\sigma'_{3n}\right]\left(s + m_b\sigma'_{3n}\right)^{a-1}}{(1+a)(2+a)\sqrt{1 + \left(6am_b\left(s + m_b\sigma'_{3n}\right)^{a-1}\right)/\left((1+a)(2+a)\right)}} \qquad [13]$$

where

$$\sigma_{3n} = \sigma'_{3\max} / \sigma_{ci}$$

Note that the value of $\sigma'_{3\max}$, the upper limit of the confinement stress on which the relationship between the criterion of Hoek-Brown and the Mohr-Coulomb one is considered, must be determined for each individual case.

The Mohr-Coulomb tangential force τ, for a given normal stress σ, is obtained by substituting these values of c' and ϕ' in the equation:

$$\tau = c' + \sigma \tan\phi' \qquad [14]$$

The equivalent plane, in terms of major and minor principal stresses, is defined by

$$\sigma_1' = \frac{2c'\cos\phi'}{1-\sin\phi'} + \frac{1+sen\phi'}{1-\sin\phi'}\sigma_3' \qquad [15]$$

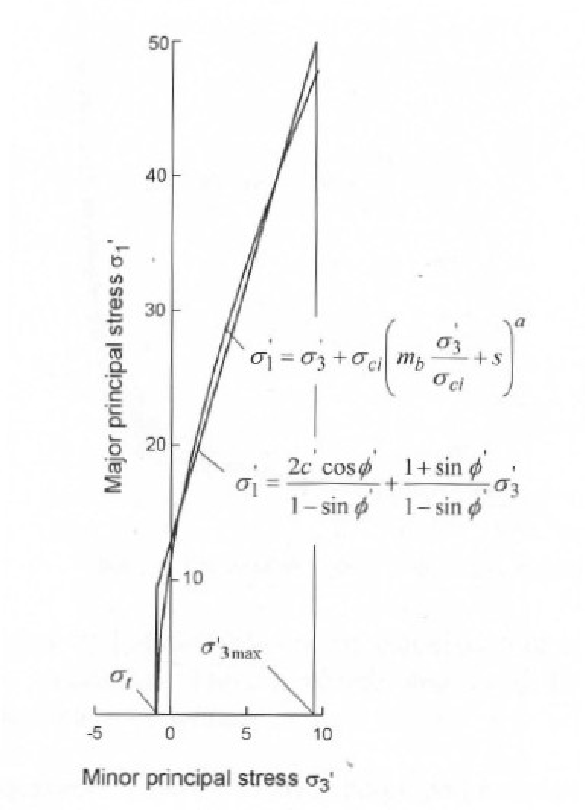


***Figure 1**: Relations between the major and minor principal stresses for Hoek-Brown and equivalent Mohr-Coulomb criterion*

The strength of the rock mass

The uniaxial compressive strength of the rock mass σ_c is given by equation [6]. The failure begins at the edge of an excavation when σ_c is exceeded by the stress induced by this limit.
The failure propagates from this initial point in a field of biaxial stress and it finally stabilizes when the local strength, defined by the equation [2], is the highest of the induced stresses σ_1' and σ_3'. Most of the numerical models are able to follow this process of failure propagation

and this level of detailed analysis is very important when considering the stability of excavations in rock and in the design of support systems.

However, there are times when it is useful to consider the general behavior of a rock mass rather than the process of failure propagation detailed above. For example, when considering the resistance of a pillar, it is more useful to have an estimate of the total resistance of the pillar, rather than a detailed knowledge of the extent of the fracture propagation in the pillar. This leads to the concept of global '*strength of the rock mass*' and Hoek and Brown have suggested that this could be estimated from the relationship of Mohr-Coulomb:

$$\sigma_{cm}' = \frac{2c'\cos\phi'}{1-\sin\phi'} \qquad [16]$$

with c' and ϕ' determined for the stress interval:

$$\sigma_t < \sigma_3' < \sigma_{ci}/4$$

giving

$$\sigma_{cm}' = \sigma_{ci}\frac{(m_b + 4s - a(m_b - 8s))(m_b/4 + s)^{a-1}}{2(1+a)(2+a)} \qquad [17]$$

Determination of $\sigma'_{3\max}$

The problem of determining the appropriate value of $\sigma'_{3\max}$ to use in the equations 12 and 13 depends on the specific application. Two cases will be studied:

1. Tunnels - where the value of $\sigma'_{3\max}$ is what gives equivalent characteristic curves for the two failure criteria for deep tunnels or equivalent settlement profiles for surface tunnels.
2. Slopes - here the calculated safety factor and the shape and location of the failure surface must be equivalent.

For the case of deep tunnels, closed-form solutions for both Generalized Hoek-Brown Criterion and Mohr-Coulomb Criterion have been used to

generate hundreds of solutions and find the value of $\sigma'_{3\max}$ which gives equivalent characteristic curves.
For surface tunnels, where the depth below the surface is less than 3 diameters of excavation, comparative numerical studies of the measurement of failure and surface settlement have given a relationship identical to what was obtained for the deep tunnels, provided that caving in the surface is avoided.

The results of the studies for deep tunnels are represented in Figure 2 and the equation inserted for both cases is:

$$\frac{\sigma'_{3\max}}{\sigma'_{cm}} = 0.47\left(\frac{\sigma'_{cm}}{\gamma H}\right)^{-0.94} \qquad [18]$$

where σ'_{cm} is the strength of the rock mass, defined by the equation 17, γ is the unit weight of the rock mass, *H* is the depth of the tunnel below the surface. In cases in which the horizontal stress is higher than the vertical one, the value of the horizontal stress should be used instead of γH.

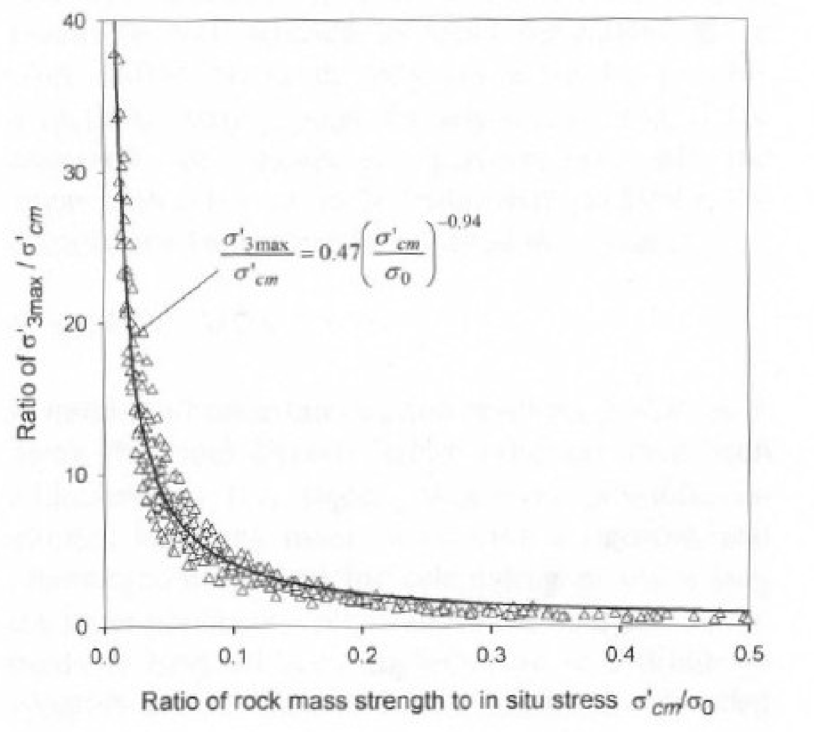


Figure 2: Report for the calculation of $\sigma'_{3\max}$ *for equivalent parameters of Mohr-Coulomb and Hoek-Brown for tunnels.*

The equation [18] applies to all underground excavations, which are surrounded by a failure zone that does not extend to the surface. For studies on issues such as the blocking of the excavation in the mines is recommended that you do not attempt to correlate the parameters of Hoek-Brown and Mohr-Coulomb, and the determination of the material properties and the subsequent analyzes are based on only one of these criteria.
Similar studies for slopes, using the circular failure analysis of Bishop (1955) for a wide range of slopes geometries and properties of rock masses have given:

$$\frac{\sigma'_{3\max}}{\sigma'_{cm}} = 0.72\left(\frac{\sigma'_{cm}}{\gamma H}\right)^{-0.91} \qquad [19]$$

where *H* is the height of the slope.

1.3 Disturbance factor D

Experience in the design of slopes in large open pit mines has shown that the Hoek-Brown criterion for rock masses in situ undisturbed (D = 0) determines properties of the rock mass that are too optimistic. The effects of the damage of loud explosion as well as stress relief due to the elimination of the overburden are disruptive to the rock mass. It is believed that the properties of the rock 'disturbed', D = 1 in equations [3] and [4], are more appropriate for these rock masses.
Lorig and Varona (2000) have shown that factors such as the lateral confinement produced by different radii of curvature of the slopes (in plan) with respect to their height also influence the degree of disturbance.
Sonmez and Ulusay (1999) have analyzed five slope failures in opened coal mines in Turkey and have attempted to assign factors to each rock mass based on their assessment of the rock mass properties predicted by the Hoek-Brown criterion. Unfortunately, one of the slope failures slope appears to be structurally controlled while another is constituted by a

stack of waste transported. The authors believe that the Hoek-Brown criterion is not applicable for these two cases.
Cheng and Liu (1990) report the results of back-analysis of very careful deformation measurements, from strain gauges placed before the start of the excavation in the cave of Mingtan in Taiwan. It was found that an area damaged by the explosion extended for a distance of about 2 m around all the big excavations. The calculated strength and the deformation properties of the damaged rock mass give a equivalent disturbance factor D = 0.7.
From these references it is clear that a large number of factors can influence the disturbance degree in the rock mass surrounding an excavation and that it could never be possible to precisely quantify these factors. However, based on their experience and on an analysis of all the details contained in these documents, the authors have tried to develop a set of guidelines for the estimation of factor D, and these are shown in Table 1.
The influence of this disturbance factor can be significant. This is shown by a typical example in which $\sigma_{ci} = 50Mpa$, $m_i = 10$ and $GSI = 45$. For an in situ undisturbed rock mass surrounding an excavation to a depth of 100 m, with a disturbance factor D = 0, the equivalent friction angle is $\phi' = 47.16°$ while the cohesive force is $c' = 0.58Mpa$. A rock mass with the same basic parameters, but in a slope more than 100 m in height, with a disturbance factor D = 1, it has an equivalent angle of friction of $\phi' = 27.61°$ and a cohesive force of $c' = 0.35Mpa$.

Table 1: Guidelines for the estimation of the disturbance D

Description of the rock mass	Suggested value of D
The excellent quality of the controlled explosion or excavation through the Tunnel Boring Machine (TBM) results into a	D=0

Description of the rock mass	Suggested value of D
minimal disturbance to the confined rock mass surrounding an excavation.	
The manual or mechanical excavation in rock masses of low quality (without the use of explosives) translates into a minimum disturbance to the surrounding rock mass. Where the compression problems are raised in the significant plan, the disturbance can be severe unless is placed a temporary basis.	D=0 D=0.5
A non controlled explosion in an excavation of hard rock causes a severe local damage, which extends for 2 to 3 m in the surrounding rock mass.	D=0.8
An explosion of small-scale cuts in	D=0.7

Description of the rock mass	Suggested value of D
embankments for civil engineering works causes modest damage to the rock mass, particularly if it is used the controlled burst. However, the release of stress causes some disturbance.	Explosive with controlled charges D=1.0 Explosive with not controlled charges
The slopes of the very large open pit mines suffer from a significant disturbance due to the heavy explosion and also due to release the stress generated by removing the overburden. In some soft rock excavation can be performed through ripping and dozing, and the degree of damage to the slope is minor.	D=1.0 Use of explosives D=0.7 Mechanized excavation

1.4 Bearing capacity of foundations on rock

The bearing capacity of shallow foundations on rock applying methods based on the failure criterion of Hoek and Brown

The verification to limit states of the "soil-foundation" complex according to the new legislation, concerns the assessment of the failure mechanisms determined by the mobilization of the overall strength of the materials. In particular, for each ultimate limit state the following condition must be respected:

$$E_d \leq R_d$$

where Ed is the design value of the action or of the effect of the action, Rd is the design value of the geotechnical system strength.
In this case, the foundations are made of rectangular elements with small size of 3.0 m amounted to a depth of 1.5 m below ground level, or directly resting on the rocky metarenite substrate.
For verification, since the foundation soils are stony rocks although fractured, reference was made to the calculation methods proposed by Carter and Kulhawy (1988) and by Serrano, Olalla and Gonzalez (2000), both based on the failure criterion of Hoek and Brown, valid just for shallow foundations on stone materials.

Method of Carter & Kulhawy

The ultimate bearing capacity of a rock mass can be written in the form:

$$q_u = \sigma_{ci} N_\sigma \qquad [20]$$

where σ_{ci} is the unconfined compressive strength of the intact rock and N_σ is defined as bearing capacity factor. According to this criterion, the ultimate bearing capacity of a rock mass is considered as a "fraction" of the uniaxial compressive strength of the intact rock.
The resolution of Carter and Kulhawy refers to the theorem of the lower limit, in which the stresses state must satisfy the equilibrium and must not violate the plasticity conditions. The resolution of the problem can be achieved by examining a load condition in which the rock mass is considered as devoid of weight and is divided into two zones; in particular in zone I, the rock being devoid of weight, the minor principal

stress σ_3 coincides with the vertical direction, while the major principal stress σ_1 coincides with the horizontal direction. In this zone the value of the principal stress σ_1 is obtained from the general equation [2] by putting $\sigma_3 = 0$ and corresponds to the unconfined compression strength resistance to compressive of the rock mass.

$$\sigma'_1 = \sigma_c = \sigma_{ci} s^a \qquad [21]$$

In zone II, ie below the foundational structure, the σ_1 is vertical and is equal to q_u (ultimate bearing capacity), while σ_3, since the equilibrium along the contact between zone I and zone II must be maintained, assumes the value shown in equation 21.

Replacing the values of σ_1 and σ_3 shown in the formula of the general failure criterion of Hoek and Brown, in correspondence of zone II is:

By simplifying and putting σ_{ci} in evidence, we obtain:

$$q_u = \left[s^\alpha + \left(m_b s^\alpha + s \right)^\alpha \right] \sigma_{ci} \qquad [22]$$

Equation [22] can be made consistent with the 20 puting:

$$N_{\sigma 0} = \left[s^\alpha + \left(m_b s^\alpha + s \right)^\alpha \right]$$

Where the symbol $N_{\sigma 0}$ indicates that the rock mass is considered to be devoid of weight. Equation [22, with the various parameters calculated according to the types of rock masses presented by Hoek and Brown, is used to produce the computation guidelines for the bearing capacity of shallow foundations in rock masses in "*AASHTO - Standard specification for highway bridges.*"

Method of Serrano, Olalla and Gonzalez

Serrano and Olalla (1998) and Serrano, Olalla and Gonzalez (2000) have proposed a method for estimating the ultimate bearing capacity of shallow foundations on rock masses. The calculation is based on the theory of slip line developed by Sokolowsky and also uses the criterion developed by Hoek and Brown. The expression that allows the computation of the ultimate bearing capacity is the following:

$$q_u = \beta_n \left(N_\beta - \varsigma_n \right) \qquad [23]$$

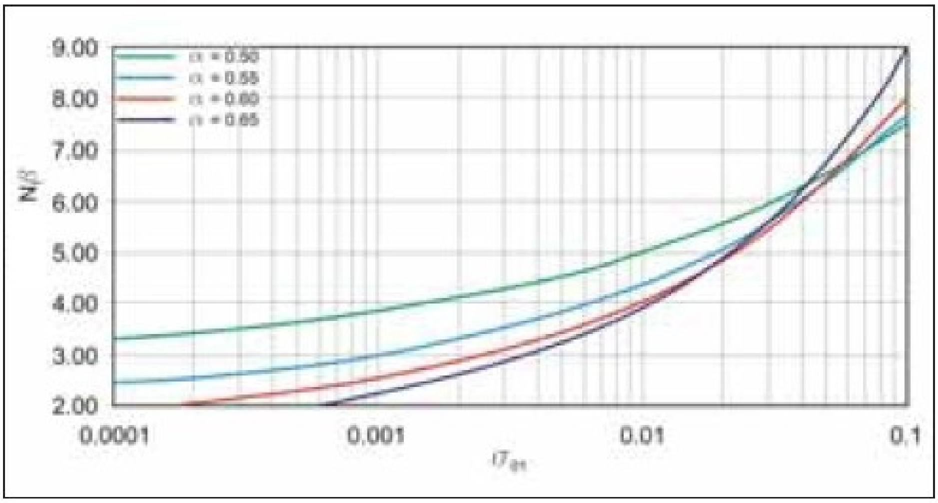
where ς_n and β_n are constants of the rock mass that depend on m_b, α , s and σ_{ci} according to the following expressions:

$$A_n = \left[\frac{m_b (1-a)}{2^{\frac{1}{\alpha}}} \right]^{\alpha/1-\alpha}$$

$$\beta_n = A_n \sigma_{ci}$$

$$\varsigma_n = \frac{s}{m_b A_n}$$

The factor N_β can be determined graphically using the abacus provided by Serrano et al. (2001), shown in the following figure:



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